

# Reliability modeling of sensors network system for critical applications

Swajeeth Pilot Panchangam  
Reliability Engineering Centre,  
Indian Institute of Technology Kharagpur,  
Kharagpur, West Bengal, India.  
[swajeeth@gmail.com](mailto:swajeeth@gmail.com)

Prof. V N A Naikan  
Reliability Engineering Centre,  
Indian Institute of Technology Kharagpur,  
Kharagpur, West Bengal, India.  
[naikan@hijli.iitkgp.ernet.in](mailto:naikan@hijli.iitkgp.ernet.in)

**Abstract—** This paper focuses on improving the effectiveness of condition monitoring and health assessment of critical systems using multiple sensors. The paper proposes an approach based on the Bayes theorem for modeling the reliability of sensors which are expected to operate under multiple load conditions. The paper also proposes an algorithm for optimal allocation of number of sensors of different types such as current sensor, temperature sensor, and vibration sensor for achieving specified sensors system reliability with minimum cost. A hypothetical case of the sensors system of an electric motor is presented to illustrate the proposed approach.

**Keywords-** Sensor redundancy; reliability; condition monitoring; sensor modeling.

## I. INTRODUCTION

Reliability of sensors used for monitoring various parameters of critical systems is very important for timely assessment of their health and to take appropriate measures for fault diagnosis at incipient stages in order to prevent any catastrophic failures. This paper focuses on modeling of reliability of sensors under multiple load conditions and optimization of sensor system reliability by application of multiple sensors. This type of modeling can be useful for applications such as propulsion systems of a satellite, nuclear power plants, aircraft systems etc. Such systems require continuous reliable monitoring system to avoid unexpected failures which might result in huge economic losses apart from ill effects on environment, health & safety of human beings and other species. Instead of spending huge amounts on replacement/repair of industrial systems due to unreliable sensors it may be better to have a high reliable sensor system with adequate redundancies. An attempt is made in this paper to develop a model for evaluation of sensor reliability under multiple loading conditions. An algorithm is also proposed to find out the best combination of each type of sensors to achieve specified sensor system reliability with minimum cost.

The rest of the paper is organized as follows:

In Section-II, the problem statement and modeling approach for reliability evaluation of sensors under two operating conditions such as normal load and full load of an electric motor is presented. In Section-III, various steps of the proposed algorithm are presented. A methodology for

evaluating the combined output of redundant sensors is proposed in the section IV. Conclusions are presented in section-V followed by selected references.

## II. RELIABILITY MODELING OF SENSORS

### A. Problem statement

Operational health of electric motors can be assessed by monitoring its' critical parameters such as current, temperature, and vibration with the help of sensors. These sensors are expected to operate reliably in two operating load conditions of the motor (discussed in B). Correct assessment of motor health can only be achieved if highly reliable redundant sensors are used for monitoring. Several algorithms are being used for evaluating the optimum number of sensors to meet the required target sensors system reliability. Refer previous version of cost minimization algorithm in the paper [1]. Since the 1950s [4], several studies have been devoted to this problem and many papers have been published on this particular topic.

This paper is focused on two aspects of this important problem. First research objective is to develop a methodology for reliability modeling of individual sensor under different operational states of an electric motor such as normal load and full load. Second objective of this work is to develop an algorithm for optimal allocation of the sensors to meet the given system target reliability.

### B. Modeling approach

In this section we propose a model for evaluating the reliability of each sensor. For this purpose we need to establish the expected range of outputs of each sensor under normal & full load conditions as shown in tables I & II respectively. For example consider an electric motor with three sensors as shown in fig.1. These sensors are in series configuration as shown in fig.2. As discussed earlier, the motor is expected to operate under the following load conditions.

a) Motor driving a load such that motor's critical parameters' values lies in the range defined in table I (Normal load mode).

b) Motor driving a load such that motor's critical parameters lies in the range as defined in table II (Full load condition).

After establishing reliability of each sensor, we propose an algorithm to optimally allocate redundancies to the sensors so as to meet a given target sensor system reliability with minimum cost. The proposed methods are presented below.

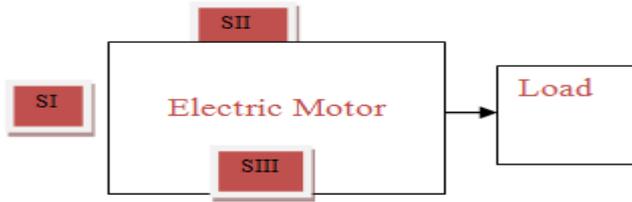


Fig.1.Sensors position on an electric motor

SI: current sensor  
SII: Temperature sensor  
SIII: Vibration sensor

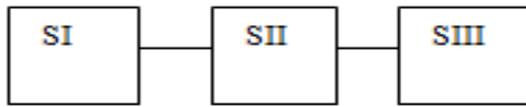


Fig.2. Functional wise series configuration of three sensors for motor monitoring

### C. Steps for Modeling approach:

- 1) Run the motor in normal load condition and collect the corresponding sensors outputs. Repeat the experiment several times.
- 2) Collect similar data under full load condition.
- 3) Find best fitting distribution for these data. The paper [3] suggests using a normal distribution for this.
- 4) Evaluate probabilities that the sensors outputs are within established threshold limits (tables I & II) for both load conditions.
- 5) Evaluate reliabilities of each sensor by applying Bayes theorem considering both the load conditions.
- 6) Development of algorithm for optimal allocation

Notations

$X_{CN}$  Motor current consumption in Normal load Condition

$X_{CF}$  Motor current consumption in full load Condition

$X_{TN}$  Motor temperature in normal load condition

$X_{TF}$  Motor temperature in full load condition

$X_{VN}$  Motor vibration in normal load condition

$X_{VF}$  Motor vibration in full load condition

$Y_{CN}$ ,  $Y_{CF}$ ,  $Y_{TN}$ ,  $Y_{TF}$ ,  $Y_{VN}$ , and  $Y_{VF}$  represent the similar quantities for current, temperature, and vibration sensors in normal and full load conditions respectively.

$R_C$  Reliability of current sensor

$R_T$  Reliability of temperature sensor

$R_V$  Reliability of vibration sensor

$R_S$  Total sensors network system reliability

$R_G$  Target reliability/ Reliability goal

$C_S$  Total sensors network system cost

$C_C$  Cost of current sensor

$C_T$  Cost of temperature sensor

$C_V$  Cost of vibration sensor

$MC$  maximum possible cost of a sensors system

$S_C$  Configuration of sensors network system; a, b, c

$N$  Maximum number of each type of sensors can be Used in a network

Variables; a=1, 2... N for current sensors, b=1, 2.....N for temperature sensors, and c=1, 2.... N for vibration sensors

Reliabilities of current, temperature, and vibration sensors are obtained by using Bayes theorem in Probability theory [5] as below.

$$R_C = P\left(\frac{Y_{CN}}{X_{CN}}\right) * P(X_{CN}) + P\left(\frac{Y_{CF}}{X_{CF}}\right) * P(X_{CF}) \quad (1)$$

$$R_T = P\left(\frac{Y_{TN}}{X_{TN}}\right) * P(X_{TN}) + P\left(\frac{Y_{TF}}{X_{TF}}\right) * P(X_{TF}) \quad (2)$$

$$R_V = P\left(\frac{Y_{VN}}{X_{VN}}\right) * P(X_{VN}) + P\left(\frac{Y_{VF}}{X_{VF}}\right) * P(X_{VF}) \quad (3)$$

TABLE I: THRESHOLDS DEFINED FOR SENSORS WHEN MOTOR OPERATING IN NORMAL LOAD CONDITION (HEALTHY)

S.No	Sensor type	Threshold		units
		LL	UL	
1	Current	$C_{LN}$	$C_{UN}$	amps
2	Temperature	$T_{LN}$	$T_{UN}$	$^{\circ}C$
3	Vibration	$V_{LN}$	$V_{UN}$	g

TABLE II: THRESHOLDS DEFINED FOR SENSORS WHEN MOTOR OPERATING IN FULL LOAD CONDITION (UNHEALTHY)

S.No	Sensor type	Threshold		units
		LL	UL	
1	Current	$C_{LF}$	$C_{UF}$	Amps
2	Temperature	$T_{LF}$	$T_{UF}$	$^{\circ}C$
3	Vibration	$V_{LF}$	$V_{UF}$	G

UL: upper limit LL: Lower limit

For several systems which requires operation with high reliability, operational safety, and availability: multiple sensors can be installed for improving the reliability of sensors network system. For such applications, the system designer

needs to evaluate the minimum number of each type of sensor. In practical situations we need to optimize the sensors network system reliability under several constraints such as cost, volume, weight, and space etc. In this paper, an algorithm is proposed for redundancy optimization of the sensors network system. The proposed algorithm is as follows:

Objective function: Minimize cost of sensor system

Constraint: Target reliability

Decision variables: Number of sensors of each type to be installed.

Mathematically, this can be formulated as follows:

Objective function: Minimize  $C_S$

Constraint:  $R_S \geq R_G$

Decision variables: a, b, c

As an example for the system under consideration (motor with three sensors, as shown in fig.1.) we get the following system of equations.

Objective function:

Minimization of total cost of sensors system,

$$C_S = (a \times C_C) + (b \times C_T) + (c \times C_V) \quad (4)$$

The sensors network system can be expressed with following Reliability Logic Diagram. It can be seen that they are in parallel series configuration; this configuration can be used for reliability modeling of sensors network system as follows

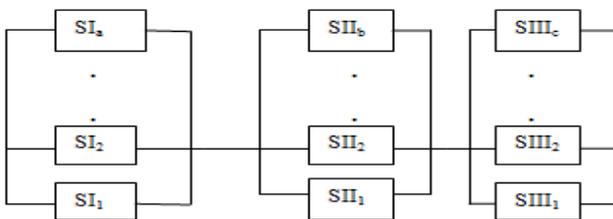


Fig.3. Reliability logic diagram of sensor network system

$$R_S = [1 - (1 - R_C)^a] \times [1 - (1 - R_T)^b] \times [1 - (1 - R_V)^c] \quad (5)$$

Constraint:  $R_S \geq R_G$

Decision variables: a, b, and c

It can be seen that the decision variables are integers and constraint equation is non-linear.

∴ A non-linear integer programming to be used for finding optimal allocation to achieve given system target reliability.

The methodology used in MIP algorithm can be suitably modified to solve this problem. It is worth mentioning here, in MIP algorithm:

Objective function: Maximization of  $R_S$

Constraint: Budget  $\leq C_S$

Proposed algorithm for finding the optimal configuration is presented in the next section.

### III. PROPOSED ALGORITHM AND ITS' APPLICATIONS

The main steps are as follows:

1. Set a = b = c = 1
2. Read  $R_C$ ,  $R_T$ ,  $R_V$ ,  $C_C$ ,  $C_T$ , and  $C_V$
3. Read  $R_G$  and N
4. Evaluate MC
5. Perform  $c = c + 1$
6. If  $c \leq N$ , go to step 7 else go to step 11
7. Evaluate  $R_S$ ,  $C_S$ , &  $S_C$
8. If  $R_S \geq R_G$ , go to step 9 else go to step 5
9. If  $C_S \leq MC$ , go to step 10 else go to step 5
10.  $MC = C_S$ , store  $S_C$  and go to step 5
11. Perform  $b = b + 1$
12. If  $b \leq N$ , make  $c = 1$  and go to step 7 else go to step 13
13. Perform  $a = a + 1$
14. If  $a \leq N$ , make  $c$  &  $b = 1$  and go to step 7 else go to step 15
15. Display  $R_S$ ,  $C_S$ , &  $S_C$

Proposed algorithm can also be used for redundancy allocations under various other situations such as Minimization of energy consumption, size, volume, and weight of a sensor system.

### IV. COMBINING OUTPUTS OF SENSORS IN EACH NETWORK

The following methodology is proposed to combine the outputs of each type of sensor network with redundancies.

For current sensor network, net output (decision) = Average of all output values of sensors in a network I.e.  $\frac{x_1 + x_2 + \dots + x_a}{a}$ . Where  $x_1, x_2, \dots, x_a$  representing outputs of set of all current sensors in a network.

For temperature sensor network, net output (decision) = Average of all output values of sensors in a network I.e.  $\frac{x_1 + x_2 + \dots + x_b}{b}$ . Where,  $x_1, x_2, \dots, x_b$  representing outputs of set of all temperature sensors in a network.

For vibration sensor network, net output (decision) = Average of all output values of sensors in a network I.e.  $\frac{x_1 + x_2 + \dots + x_c}{c}$ . Where,  $x_1, x_2, \dots, x_c$  representing outputs of set of all vibration sensors in a network.

Finally, decision is taken based on average values obtained from current, temperature, and vibration sensor networks whose values falls in one of the values defined in tables I & II.

**Note:** For instance if algorithm outputs 4, 4, 2; configuration needed to achieve a given target reliability, then a=4 (current sensor network), c=4 (temperature sensor network), and b=2 (vibration sensor network). Maximum

number of redundancies to be added to each type of sensor is limited by the physical geometry of the motor considered for application.

**Illustrative example:** - The proposed modeling approach has been illustrated by considering an electric motor which is used as a prime mover of a centrifugal pump, as discussed earlier three types of sensors are used for monitoring the parameters: current, temperature, and vibration of the motor. As a hypothetical case to illustrate the proposed model, following data are assumed in tables III, IV, V, VI, and VII. The target reliability specified for the sensors network system is 0.989. The costs of each sensor considered are given below.

TABLE III: COST OF EACH SENSOR

Sensor type	Cost
Current	15\$
Temperature	22\$
Vibration	30\$

TABLE IV: THRESHOLDS DEFINED FOR SENSORS WHEN MOTOR OPERATING IN NORMAL LOAD CONDITION (HEALTHY)

S.No	Sensor type	Threshold		units
		LL	UL	
1	Current	3	6	amps
2	Temperature	30	90	<sup>o</sup> C
3	Vibration	2	5	g

TABLE V: THRESHOLDS DEFINED FOR SENSORS WHEN MOTOR OPERATING IN FULL LOAD CONDITION (UNHEALTHY)

S.No	Sensor type	Threshold		units
		LL	UL	
1	Current	6	9	amps
2	Temperature	90	150	<sup>o</sup> C
3	Vibration	5	8	g

TABLE VI: EXPECTED MEANS AND STANDARD DEVIATIONS OF THE MOTOR

Parameter	Normal load condition			Full load condition		
	X <sub>CN</sub>	X <sub>TN</sub>	X <sub>VN</sub>	X <sub>CF</sub>	X <sub>TF</sub>	X <sub>VF</sub>
Mean ( $\mu$ )	4.5	7.5	60	120	3.5	6.5
Standard deviation ( $\sigma$ )	1.5	1.5	30	30	1.5	1.5

TABLE VII: EXPECTED MEANS AND STANDARD DEVIATIONS OF CURRENT, TEMPERATURE, AND VIBRATION SENSORS

Parameter	Normal load condition			Full load condition		
	X <sub>CN</sub>	X <sub>TN</sub>	X <sub>VN</sub>	X <sub>CF</sub>	X <sub>TF</sub>	X <sub>VF</sub>
Mean ( $\mu$ )	4.3	7.3	57	125	3.2	6.3
Standard deviation ( $\sigma$ )	1.2	1.6	35	37	2.0	1.8

**Solution:** - following the steps mentioned in section II. (C), we get the following results

TABLE VIII: RESULT SHOWING RELIABILITIES OF CURRENT, TEMPERATURE, AND VIBRATION SENSORS

Sensor type	Reliability
Current sensor	0.84393
Temperature sensor	0.94680
Vibration sensor	0.63580

Now, for a given target reliability of 0.989, number of redundancies for each sensor can be used, and cost of each sensor; proposed algorithm gives the outputs shown in table below.

TABLE IX: RESULTS SHOWING OPTIMAL CONFIGURATION, ACHIEVED RELIABILITY, & MINIMUM COST

S <sub>c</sub>	4, 3, 5
R <sub>s</sub>	0.989228
C <sub>s</sub>	276.00\$

Now, take the averaged values of four current, three temperature, and five vibration sensors for making decision.

Case (i): If these values fall in the range of table IV, then the motor works in normal load condition.

Case (ii): If these values fall in the range of table V, then the motor works in full load condition.

The results obtained using proposed algorithm are verified using MIP algorithm [2] by giving maximum cost as the cost obtained through proposed algorithm.

## V. CONCLUSIONS

In this paper we presented a novel model to evaluate reliabilities of current, temperature, and vibration sensors. A new algorithm is proposed to evaluate the optimum configuration of sensor networks required to achieve target reliability with minimum cost. The case of an electric motor is taken for illustration of the proposed methodology. The methodology is validated by using the MIP algorithm. For better results It is proposed to take average values of the outputs of each type of sensors for decision making regarding

the operational health of the system under different load conditions. The proposed algorithm can be used for reliability analysis of sensor networks systems installed for various critical applications such as: space applications, nuclear power plants, and chemical industries etc. These critical systems require usage of multiple sensors for reliable feedback on system health.

#### REFERENCES

- [1] A. O. Charles Elegbede, Chengbin Chu, Kondo H. Adjallah, and Farouk Yalaoui, "Reliability Allocation through Cost Minimization," *IEEE Trans. Reliability*, VOL. 52, NO. 1, March 2003.
- [2] K. B. Misra and Usha Sharma, "An Efficient Algorithm To Solve Integer-Programming Problems Arising In System-Reliability Design," *IEEE Trans. Rel.*, vol. 40, no. 1, April 1991.
- [3] K. Veeramachineni, L.A. Osadciw, "Biometric sensor management: tradeoffs in time, accuracy and energy," *IEEE Trans.* Vol.3, No.4, December 2009.
- [4] S. Arold and Balanba, "Allocation of system reliability," Tech. Rep., ASD-TDR-62-20, 1962.
- [5] "Reliability and Maintainability Engineering," 'Charles E. Ebeling'. McGraw-Hill International Editions 1997.